GENERATING RANDOM NOISE

System provides both a random telegraph wave with a mean count rate of 80 Kc and Gaussian noise with a power spectrum flat between d-c and 20 Kc, for fast analog computation and especially for repetitive computer applications

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ANALOG-COMPUTER studies of random processes require generating randomly varying voltages with statistical properties to represent randomly varying variables, initial conditions and parameters.[1]

A noise generator for providing random input signals to a repetitive analog computer may require a flat power spectrum from d-c to several thousand cycles a second.

The output power spectral density may be altered to meet a given requirement by linear shaping filters; the output amplitude distribution can be shaped by diode function generators.

Gaussian-noise generation by noisy resistors,[2] diodes,[3] phototubes[3] and thyratrons in a magnetic field[4] requires elaborate regulation or monitoring of rms and d-c output levels, and also has the disadvantage of a nonuniform power spectrum at low frequencies.

Low-frequency spectral nonuniformities, including line-frequency components due to hum pickup, can be removed through sampling or demodulation and filtering of the noise generator output, which recenters a flat portion of the spectrum about d-c;[5] but the resulting noise output spectra are flat only to about 100-500 cps.

A flip-flop symmetrically triggered by a radiation detector actuated with a radioactive source yields a random telegraph signal with Poisson-distributed zero crossings (Fig. 1 inset).

If the levels $\pm E$ are accurately set by a precision limiter, the mean output is zero, whereas the mean-square, autocorrelation and power spectral density are given by

$$\phi(0) = E^2$$
$$\phi(\tau) = E^2 e^{-2\alpha|\tau|}$$
$$W(\omega) = \frac{E^2}{\alpha} \frac{1}{1 + \omega^2/4\alpha^2}$$

where α is the mean count rate of the radiation-detector output pulses. Low-pass filtering of the random telegraph signal yields Gaussian noise of zero mean, with spectral density determined by the filter characteristics. The flat-spectrum random telegraph signal is useful in its own right. The binary nature permits its direct use for random switching (binary multiplication), for example, in correlators.[6] This nature permits its direct use for randomly timed events (Poisson process) in simulations of equipment failures and queing problems such as traffic control.[7]

An earlier noise generator of this type used a self-quenched Geiger-Mueller tube as the radiation detector,[8] but this device was limited to relatively low count rates (below 500 cps) by its deionization time. The noise generator replaces the Geiger-Mueller tube with a scintillation detector (type 931A multiplier phototube) and permits count rates well in excess of 75 Kc for wideband operation.

The block diagram of Fig. 1 shows the operation of the circuit. The noise generator consists of a source of radioactive material mixed in a light-emitting phosphor and a multiplier-phototube detector whose output produces current pulses corresponding to the emission of light impulses in the scintillating phosphor. After amplification in a five-stage pulse ampli-

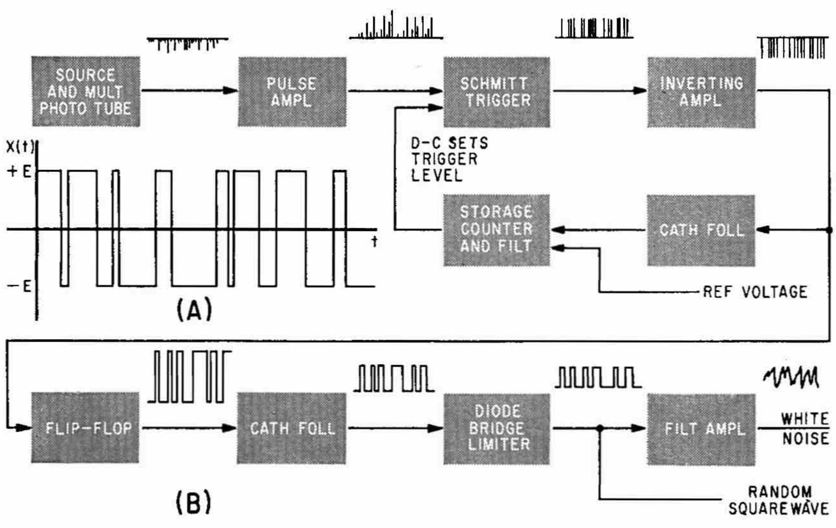

FIG. 1—Block diagram of noise generator with radioactive source; random telegraph wave (inset) with Poisson-distributed zero crossings

WITH RADIOACTIVE SOURCES

fier, pulses exceeding a threshold voltage operate a Schmitt trigger to produce short pulses of constant amplitude and random time occurrence. These pulses are inverted and applied to a flip-flop to produce a squarewave of constant amplitude with random zero crossings. The output squarewave has a d-c component, which is eliminated by a cathode follower followed by a symmetrical limiter. The cathode follower output is clipped at equal positive and negative voltage levels by a diode-bridge limiter. The result is a random telegraph signal having accurate levels of $+E$ and $-E$ volts. The random telegraph signal is amplified and filtered in a chopper-stabilized wide-band operational amplifier. The power spectral density will be flat from zero to near the filter cutoff frequency.

Since the mean count rate of the random telegraph signal affects the spectral density of the noise output, the threshold level of the Schmitt trigger is controlled by feedback through a storage counter using an operational-amplifier integrator.

The radioactive source consists of a radioactive isotope mixed with a light-emitting phosphor, such as is used on dials of luminous watches. This jewelers' paint is applied to a small region of the tube envelope covering the cathode of the multiplier phototube.

The 931A multiplier phototube was selected because of its relatively low cost, and because it is of the vacuum type, which, unlike a gas phototube, suffers little or no loss of sensitivity with use when operated continuously near full plate current.

A supply of 1,000 volts is applied to the phototube across a voltage divider providing 100 volts between successive dynode stages. A curve of count rate versus voltage across the phototube is shown in Fig. 2A for a Schmitt trigger threshold level set near maximum count rate.

The source and multiplier phototube are completely enclosed by a Mu-metal shield to prevent a-c fields from modulating the count rate; the earth's field might also cause a decrease in signal. The pulse output is of negative polarity and varies from near zero to ten or fifteen volts.

The count rate is determined by the number of multiplier phototube pulses that exceed a threshold value. The pulses occur at random times and can be close together. Hence, a fast amplifier is required, to amplify the small pulses above the threshold value, and not be overloaded by the larger pulses. The circuit is common to nuclear physics experimentation[9] (Fig. 2B). The feedback loops and design are similar to the Oak Ridge-Fairstein and Brookhaven-Chase circuits.[9] The negative signal from the multiplier phototube is amplified through V_{1A} and appears as a positive signal at the grid of V_{1B}. Tubes V_{1B} and V_{2A} are connected as a difference amplifier, with their grids coupled through a long time constant (220,000 ohms and 0.1 μf). Under normal conditions, that is, when the positive input signal at the grid of V_{1B} is small, the difference amplifier can operate with low effective cathode impedance (13,000 ohms in parallel with the r_p of the other tube) since both V_{1B} and V_{2A} are conducting. When the signal is large, V_{2A} cuts off and all current flows through V_{1B}, the cathode impedance of which is now 13,000 ohms. For large signals V_{1B} draws grid current; but since there is no coupling capacitor in the grid circuit, it will return to its quiescent operating point as soon as the large signal ceases. Tube V_3 is an inverting amplifier whose output is the triggering signal for the Schmitt trigger.

Two feedback loops help to stabilize the gain of the pulse amplifier. High-frequency negative feedback is applied from the cathode of V_{2B} to the cathode of V_{1A} through a capacitor and resistor in shunt. The capacitor in shunt with the feedback resistor overcomes the capacitance from the cathode of V_{1A} to ground. There is also a second, low-frequency feedback connection from the cathodes of V_{1B} and V_{2A} to the

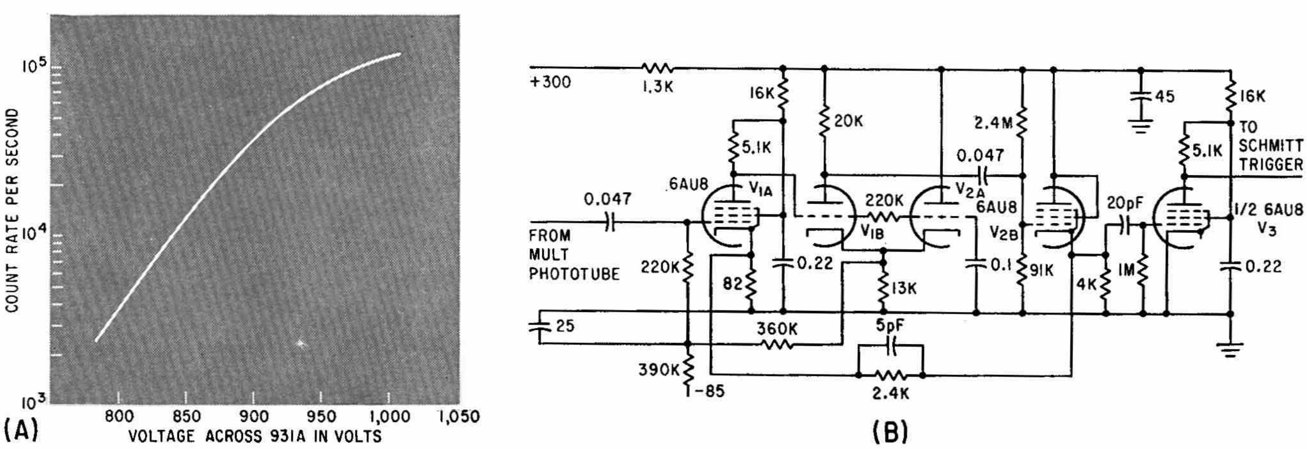

FIG. 2—Graph (A) of count rate versus voltage across the multiplier phototube; nonoverloading pulse amplifier (B)

grid of V_{1A} to stabilize low-frequency fluctuations in voltage caused, for example, by changes in tube characteristics. The long time constant of the 390,000-ohm resistor and 25-μf capacitor confines the effectiveness of this feedback to the quiescent current conditions of V_{1A} and the difference amplifier.

Tests were made on the amplifier with an astable multivibrator at 2 Mc. The rise time of the input pulse was 0.04 μsec, and the recovery time was 0.16 μsec. Amplifier rise time is no greater than 0.22 μsec; the recovery time 0.16 μsec. The pulse gain varies with input amplitude, due to the nonlinear difference amplification. The output voltage never exceeds 60 volts for an input of 4 volts. Reliable amplification is obtained in excess of 2.5 Mc.

The Schmitt trigger is of conventional design.[10] An output amplifier inverts the trigger and prevents loading of the trigger by the binary.

A resistance in the cathode circuit of the first tube reduces the loop gain to near unity and thus reduces hysteresis to about 1.8 volts. If the hysteresis were eliminated entirely, the loop gain would be equal to unity, which is a relatively unstable condition. Drift due to supply voltage changes or tube aging could cause negative hysteresis (loop gain less than unity) and the circuit would no longer change states.[10] The +1.8-volt hysteresis will ensure that the loop gain remains greater than unity even if the circuit drifts. The rise time depends on the speed of the triggering voltage, but is in no case less than 0.16 μsec. The output switches between +250 and +300 volts with a recovery time of 0.18 μsec. The maximum reliable count rate using the astable multivibrator connected to the input of the pulse amplifier is about 2.6 Mc. Drift of the switching point is eliminated by the feedback control circuit.

The flip-flop is also of conventional design.[10] An output cathode follower reduces the output voltage to near the desired level, lowers the output impedance and prevents loading the binary.

The output of the cathode follower is slightly greater than ±30 volts, as the diode bridge will limit to ±20 volts. This 20-volt margin ensures the elimination of any unwanted noise riding on the random squarewave, and also results in sharper pulses. Since the random squarewave is also attenuated in the resistance network preceding the cathode follower, the flip-flop output must be large. Furthermore d-c coupling is used from the flip-flop to the noise generator output to preserve the components of the random signal down to d-c. The cathode follower also acts as an isolation network; it prevents the diode bridge from loading the flip-flop, and at the same time feeds the bridge through a low input impedance.

The binary output swings from +300 to +180 volts or an output of 120 volts. The rise time is 0.6 microsecond and the recovery time is 0.3 μsec. The maximum number of reliable transitions is 0.7 million a second. The output of the cathode follower is from +30 to −30 volts.

To obtain an output voltage of adequate root mean square value with a stable zero mean, the output cathode follower is followed by the diode-bridge limiter[11] (Figs. 1A and 1B). To allow fast rise times, the impedance level should be as low as possible. Precision carbon deposited resistors provide accurate limiting levels. The bridge-bias voltages must be well regulated and preferably should be obtained directly from the reference supplies of the computer in which the device will be used. The circuit limits at ±20 volts within 0.3 volts for both short and long pulses.

The output element of the complete random noise generator must be a wide-band, d-c amplifier capable of supplying the required gain and low output impedance. A Philbrick USA-3 chopper-stabilized amplifier is used. The output low-pass filter is placed in the feedback loop of this amplifier; plug-in terminals permit use of different feedback networks.

The count rate can be set by the threshold level at the Schmitt trigger input. Figure 4A shows the feedback circuit. The count rate was measured at the output of the inverting amplifier.

Count-rate regulation could also be accomplished through feedback to the power supply of the phototube. This would have the advantage of having more elements within the control loop. This

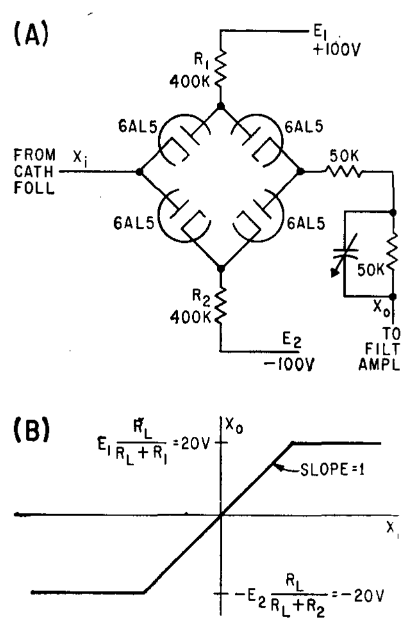

FIG. 3—Diode-bridge limiter (A) and its transfer characteristic (B)

method is not used because other phototubes may use the same power supply.

Referring to Fig. 4B, cathode follower V_5 applies negative pulses through a low impedance to the storage counter. The negative pulses cause capacitor C_1 to charge through diode D_1. The time constant with which C_1 charges is the product of C_1 times the sum of the diode and cathode follower resistances. If this time constant is small compared with the duration of the pulse, then C_1 will charge fully to the value $e_1 = E$, with the polarity indicated. During the charging time of C_1, diode D_2 does not conduct and no current flows into the operational amplifier. At the termination of the input pulse, capacitor C_1 is left with the voltage $e_1 = E$, which now appears across D_1 and across the series combination of D_2 and the operational amplifier. The polarity of this voltage is such that D_1 will not conduct. Capacitor C_1 will, however, discharge through D_2 into the operational amplifier. The time constant with which this transfer of charge takes place must be small in comparison with the interval between pulses, to allow equilibrium to be established between the capacitor voltages (C_1 and C_2). Capacitor C_2 is large in comparison with C_1.[10] The average current flowing into the operational amplifier is EC_1a. The node equa-

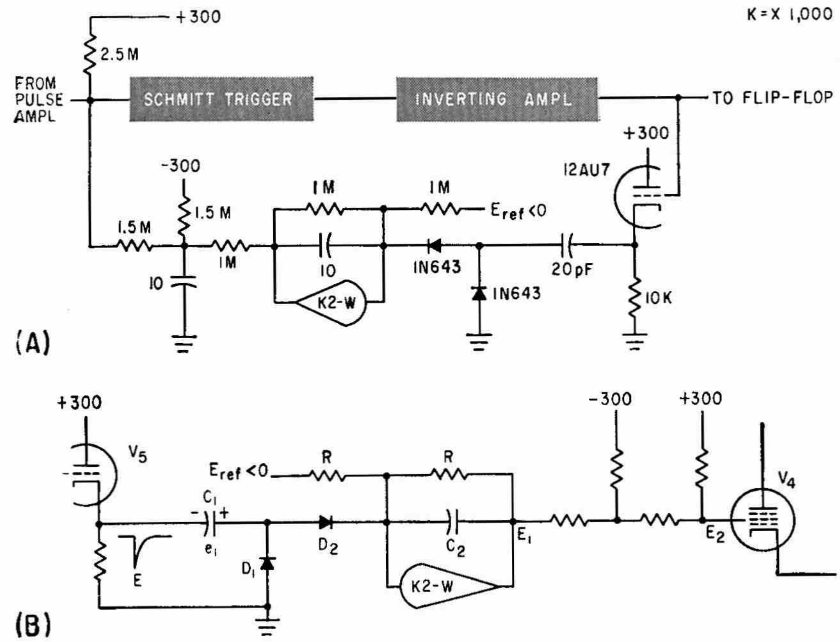

FIG. 4—Mean count rate control circuit (A), with a Philbrick K2-W operational amplifier; storage counter (B) with an operational amplifier circuit

tion at the input of the operational amplifier is $EC_1\alpha = -E_1/R$ at d-c. Solving for E_1, $E_1 = -ERC_1\alpha$.

This last equation indicates the direct dependence of the operational-amplifier output voltage on the mean count rate. If the mean count rate increases, the output voltage decreases and lowers the threshold voltage at the input of the Schmitt trigger, causing the count rate to decrease, and in doing so, regulating the noise generator.

Threshold level can be adjusted by varying the reference voltage at the operational-amplifier input.

The performance of the control circuit is

E_1 (volts)	E_2 (volts)	ΔV of Phototube (volts)
−10	+96	0
+5	+100	−50
+22	+103	−100

The count rate was varied by decreasing the voltage across the multiplier phototube. The large changes were for illustration and would never be expected to occur in practice. The reference voltage was 100 volts.

The noise generator has been built and will be incorporated in a repetitive computer now being built in the computer laboratory.[12, 13] It will be used in random-process studies.

The frequency capabilities of the generator circuits meet the design requirements and exceed the count rates obtainable from the source used. This permits use of a source of higher frequency should one become available, and provisions have been made for using two sources in parallel. If the sources are identical and independent, the mean count rate of the noise output should double.[14]

A primary consideration in random noise generator design is freedom from periodic components. The pulse circuit generating the random telegraph signal discriminates against line-frequency hum pickup, although stray 60-cps fields and currents can under some circumstances modulate the mean count rate both in the multiplier phototube and at the Schmitt input.

The effects on second-order statistics, such as the power spectrum, appear to be removed when the random telegraph signal is filtered for gaussian output.

It is believed that the main periodic interference in the random output is due to line-frequency components in the output amplifier.[15] This is due mainly to chopper ripple and spikes, which could be minimized by omission of the chopper stabilizer, and by d-c operation of the output amplifier filaments.

Small line-frequency components of this type are, unfortunately, rampant through most existing analog computers and measure about 3 to 5 mv rms. They are negligible in computations over the usual ±100-volt range, but might add up in statistical work.

In the application of the noise generator to the new repetitive analog computer, this has been taken into account; precision crystal-oscillator clock control[13] prevents computer repetition rates and sampling frequencies harmonically related to the line frequency, so that line-frequency components are averaged out in all presently projected random-voltage measurements.

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